Frontiers in Optics + Laser Science 2022

Intuitive understanding of the connection between Pancharatnam-Berry optical beam deflectors and polarization volume holograms

Fenglin Xi, Philip Bos

Department of Physics, Advanced Material and Liquid Crystal Institute, Kent State University, Kent, Ohio 44242, USA

Introduction

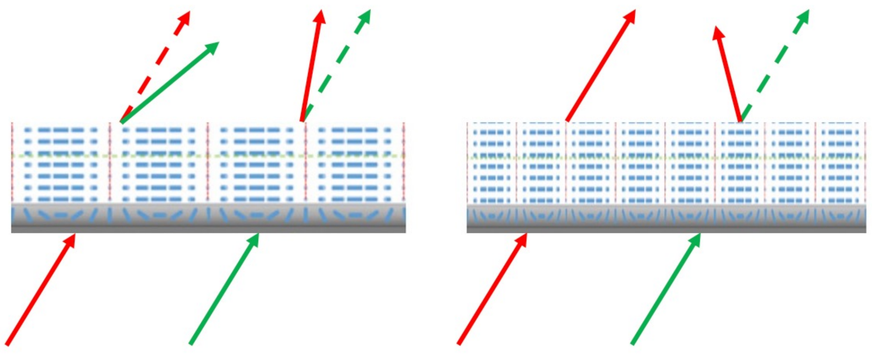

Fig. 1. Comparison of response of a PPD(left) and PVH(right).The intensity of light in the direction of the dashed arrows is close to zero when the device thickness D= $\lambda/(2\Delta n)$(λ is vacuum wavelength and Δn is birefringence).

The polarization state of light represented by the red arrow is RHC, while the green arrow is LHC. For PPD, with RHC incidence the 0th and -1st diffraction orders are observed. With LHC incidence, the 0th and $+1^{st}$ diffraction orders are observed. For PVH, with RHC incidence only 0th diffraction order is observed. With LHC incidence, the 0th and $+1^{st}$ diffraction order are observed.

All effects can be quantitatively modeled using the rigorous coupled wave approach (RCWA) or by using the finite difference time domain (FDTD) method. But these computational approaches do not provide an intuitive analytical connection between the PPD and the PVH type devices, or a predictive analytical expression to explain their properties.

For isotropic materials, Gaylord and Moharam [1] considered a similar question in exploring the connection between Raman-Nath and Bragg type devices. They used an approximate coupled wave analysis to provide an intuitive derivation of the 'p' parameter, which is used for distinguishing the Raman-Nath grating and Bragg grating effects [2]. In this work, the approach of Gaylord is extended to birefringent materials using a beam propagation method to clearly show the analytical connection between the optical properties of PPD and PVH devices.

Approximate coupled wave approach for birefringence material

Step 1: Find the form of the **dielectric tensor** → Step 2: Find the form of **electric field** → Step 3: **Plug** the electric field and dielectric tensor **into wave equation** → **Discretize** the first order differential equation to yield the propagation

Following the steps, the equation that relates the values of complex wave amplitude for the l_{th} diffraction order S_l in a layer, to the next one (layer+1) complex wave amplitude for the l_{th} diffraction order $S_l^{layer+1}$, and the incident angle is close to 0 degree(normally incidence), in the light propagation direction is,

$$S^{layer+1}{}_{-1} = S^{layer}{}_{-1} - \frac{i\pi d}{2A_{-1}}\left\{\frac{-4}{\Lambda^2}S_{-1}{}^{layer} + \frac{\Delta N^2}{2\lambda^2}M_{-}S^{layer}{}_{0}\right\} \quad \text{Equation 1}$$

$$S^{layer+1}{}_{0} = S^{layer}{}_{0} - \frac{i\pi d}{2A_{0}}\left\{\frac{\Delta N^2}{2\lambda^2}M_{-}S^{layer}{}_{1} + \frac{\Delta N^2}{2\lambda^2}M_{+}S^{layer}{}_{-1}\right\} \quad \text{Equation 2}$$

$$S^{layer+1}{}_{+1} = S^{layer}{}_{+1} - \frac{i\pi d}{2A_{1}}\left\{\frac{\Delta N^2}{2\lambda^2}M_{+}S^{layer}{}_{0}\right\} \quad \text{Equation 3}$$

We can see the energy only transfers between adjacent orders. Initially, all energy is in the 0th order.

Difference in polarization response of PPD and PVH devices

When the incident polarization state is LHC, use (1),

$$S_{-1}{}^{layer+1} = S_{-1}{}^{layer} - \frac{i\pi d}{2A_{-1}}\left\{\frac{-4}{\Lambda^2}S_{-1}{}^{layer} + \frac{\Delta N^2 M_{-}S^{layer}{}_{0}}{2\lambda^2}\right\}$$

$$= S_{-1}{}^{layer} + \frac{i\pi d}{2A_{-1}}\left\{\frac{-4}{\Lambda^2}S_{-1}{}^{layer}\right\} - \frac{i\pi d A_0 \Delta N^2}{4\lambda^2 A_1}\begin{bmatrix}1 & i & 0\\ i & -1 & 0\\ 0 & 0 & 0\end{bmatrix}\begin{bmatrix}1\\ i\\ 0\end{bmatrix} = S_{-1}{}^{layer} + \frac{i\pi d}{2A_{-1}}\left\{\frac{-4}{\Lambda^2}S_{-1}{}^{layer}\right\} - \frac{i\pi d A_0 \Delta N^2}{4\lambda^2 A_1}\begin{bmatrix}0\\ 0\\ 0\end{bmatrix}$$

We can see here that there is no energy flowing into -1st order from 0th order.

Use (3),

$$S_{1}{}^{layer+1} = S_{1}{}^{layer} - \frac{i\pi d}{2A_{1}}\left\{\frac{\Delta N^2 M_{+}S^{layer}{}_{0}}{2\lambda^2}\right\} = S_{1}{}^{layer} - \frac{i\pi d A_0 \Delta N^2}{4\lambda^2 A_1}\begin{bmatrix}1 & -i & 0\\ -i & -1 & 0\\ 0 & 0 & 0\end{bmatrix}\begin{bmatrix}1\\ i\\ 0\end{bmatrix}$$

$$= S_{1}{}^{layer} - \frac{i\pi d A_0 \Delta N^2}{2\lambda^2 A_1}\begin{bmatrix}1\\ -i\\ 0\end{bmatrix}$$

We can see from here that for $+1^{st}$ order beam, excited from the 0th order LHC incident beam, the polarization state is RHC.

Similar calculation can be applied for RHC incidence. To summarize the polarization response,
1. LHC incidence can only activate the $+1^{st}$ order.
2. RHC incidence can only activate the -1st order.

Definition of the 'p' parameter for a birefringent media

In (1), we can see that the change in the 0th and $+1^{st}$ orders is related to their coupling with adjacent orders. But the change in the -1st order has a different dependence. It is related to its coupling with the adjacent 0th order, and to itself.
It will be shown below that, for -1st order, the ratio of the self-coupling term and the coupling to 0th order terms is significant to differentiating the PPD and PVH optical effects. The ratio is, This ratio will be designated as the 'p' parameter for birefringence material,

$$p = \frac{4*\lambda^2}{\Lambda^2 * n_{ave}{}' * \Delta n}$$

If the 'p' parameter is small, the self-coupling term is negligible, which results in the phase of the -1st order to be constant it propagates from layer to layer through the material. However, if the 'p' parameter is large, the self-coupling term is not negligible. In this case the phase of the -1st order changes when propagates from layer to layer, and its phase is changing with respect to the other orders.

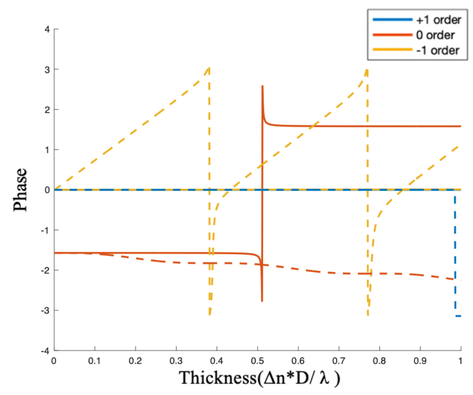

Fig. 2. Phase of p=0.012(solid line) and 9.3(dashed line)

In Fig.2, when light propagating though the thickness of the grating, with small p parameter, the phase of the -1st order has constant phase(yellow solid curve). When p parameter is large, the phase of the -1st order is rapid changing.(yellow dash curve).

References

1. Gaylord, T., and M. Moharam. "Planar dielectric grating diffraction theories." Applied Physics B 28 (1982): 1-14.
2. M. G. Moharam and L. Young, "Criterion for Bragg and Raman-Nath diffraction regimes," Appl. Opt. 17, 1757-1759 (1978)

Conclusions

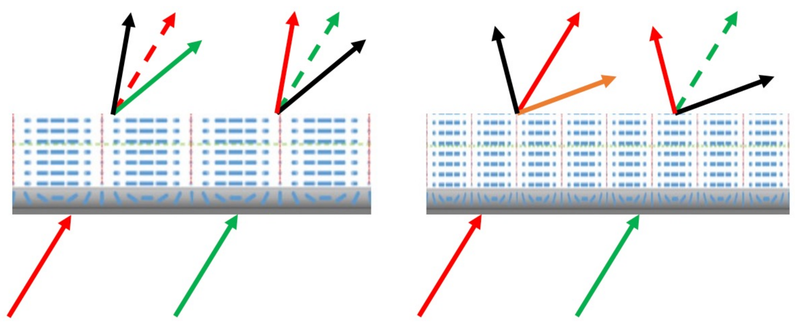

Fig. 3. Explanation of different optical responses of a PPD(left) and PVH(right).

Energy transfer to the order indicated by the solid black line is prevented because the order is not activated by the incident polarization state. Energy transfer to the order indicated by the orange line is prevented because the rapid changing phase relationship between this order and the 0th order incident beam. If the device thickness is optimized, (D= $\lambda/(2\Delta n)$), there is no energy transfer to the orders represented by dashed vectors.

Intuitive explanation

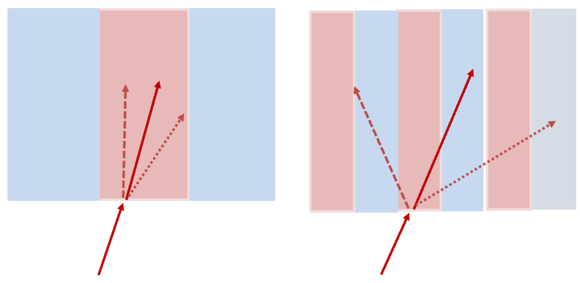

Fig.4. Figure shows an incident light beam to a periodic media, and the resulting diffracted orders for PPD(left) and PVH(right).

For PPD, because of the grating period is large, there is no difference for the $+1^{st}$ (long dashes) and -1^{st}(dotted line)order beam.

For PVH, with a much smaller grating period, the 0th order deflected beam (solid line) , and the $+1^{st}$ order beam traverse regions of similar index of refraction and are expected to have a similar phase variation as they propagate through the media. However, the -1st order sees a greater variation in the index of refraction , and therefore is expected to have a changing phase relationship to the 0th order beam.