Check for updates

Intuitive Understanding of the Limitation of Pancharatnam–Berry Optical Beam Deflectors

Fenglin Xi, Hsienhui Cheng**, Xiayu Feng**, Lu Lu**, Philip Bos**
***Liquid Crystal Institute, Kent State University, Kent, OH, 44240, USA **Reality Labs, Redmond, Washington 98052, USA**

Abstract
An approximate beam propagation method is proposed as an intuitive simulation of the optics of the Pancharatnam–Berry phase (PPD). Using this method, the limitation of PPD in diffraction efficiency for large diffraction angle is made clear.

Author Keywords
Pancharatnam–Berry phase, rigorous coupled wave approach, ρ parameter, diffraction angle, diffraction efficiency.

1. Introduction
Pancharatnam–Berry phase devices (PPD), proposed in [1], have broad applications in many optical systems. Particularly, as a lightweight and thin optical component that changes light propagation with potentially high efficiency and polarization selectivity[2][3], PPD recently attracted much interest in the AR/VR field[4].

In PPD, the liquid crystal director is in the plane of the layer and spirals about the layer normal with a pitch P_x. The birefringence Δn is the refractive index along the long ne, minus the refractive index along the short axis no(which is, Δn=ne-no).

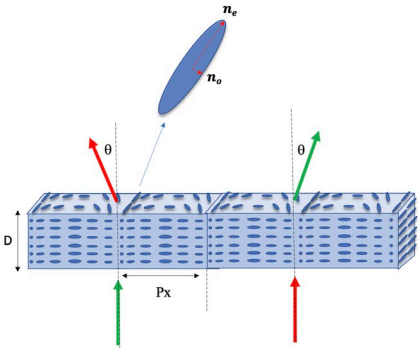

Figure 1. Liquid crystal director picture for PPD. Left-hand circular light (green) is deflected to one direction with high efficiency. Right-hand circular light (red) is deflected to the opposite direction. This picture shows a 20-degree PPD deflector.

The deflection efficiency is a function of thickness D. λ is the vacuum wavelength of the incident light. While D= λ/(2Δn), its deflection efficiency is the highest. The thickness will remain to meet this half-wave condition for the rest of this paper.

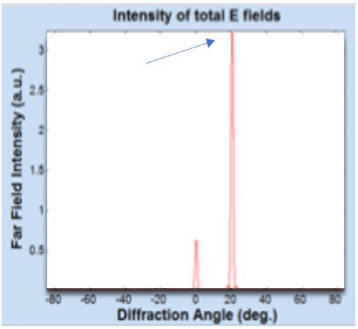

Figure 2. An example of FDTD far-field intensity result is shown when incident wavelength λ=0.633 microns, incident angle =0°, and P_x=1.85 microns.

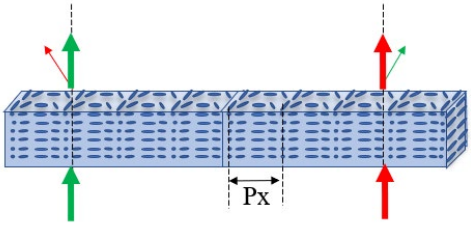

Figure 3. Liquid crystal director picture for a 40-degree PPD deflector. The P_x is smaller due to the larger diffraction angle, compared to the structure in Figure 1.

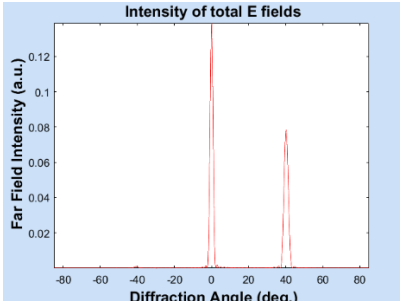

Figure 4. An example of FDTD far-field intensity result is shown when incident wavelength λ=0.633 microns, incident angle =0°, and P_x=0.98 microns.

PPD is known to be a good small angle deflector(<10 degrees) with nearly 100% efficiency. Figure.2 shows the far field intensity for a 20-degree PPD deflector. We can see that there is a noticeable drop in diffraction efficiency. For a 40-degree deflector, as in Figure.4, the PPD has less than 50% diffraction efficiency, so it is no longer a good candidate for this larger diffraction angle.

To summarize, the diffraction efficiency drops as the diffraction angle increases(P_x decreases, effectively) for PPD. This optical phenomenon can be quantitatively modeled using numerical calculations, including the rigorous coupled wave approach (RCWA) and the finite difference time domain (FDTD) method(as we have seen in Figure 2 and 4). However, these computational approaches are not transparent, like a black box. The output light information is given without explaining how the light interacts with PPD, which gives this dependence of the diffraction efficiency in diffraction angle(P_x, effectively).

It has been well-studied that the efficiency of a beam deflector made of isotropic material changes with the value of P_x. For isotropic materials, Gaylord and Moharam [5] considered a similar question in exploring the connection between Raman–Nath and Bragg type devices. For isotropic material, "ρ" parameter is used for distinguishing the Raman–Nath grating and Bragg grating effects. Oh and Escuti[6] uses "ρ" parameter as an

ISSN 0097-996X/24/5501-0161-$1.00 © 2024

analogy to predict the diffraction behavior of a PPD. In the previous paper[7], we extended the approach of Gaylord to birefringent materials using a beam propagation method to clearly show the analytical connection between the optical properties of PPD and polarization volume hologram[8] devices.

However, the analytical explanation of why diffraction efficiency drops for larger diffraction angle PPD has not yet been proposed.

In this paper, the beam propagation method in [7] is used to explain the limitations of the diffraction efficiency of a large diffraction angle PPD deflector analytically.

2. The approach

2.1 Overview of the calculation

In this work, as in [7], the starting point is the Helmholtz equation.

$$\nabla^2 E + k^2 \varepsilon E = 0 \quad (1)$$

To show the evolution of the amplitude and polarization states of light as it propagates in a periodic birefringent structure, light is considered to be decomposed into plane waves with defined directions corresponding to the allowed diffraction orders under the Floquet condition. Further, the periodic birefringent layer is considered to be discretized into layers that are perpendicular to the plane of the film. The grating is considered to consist of N layers, each with a very small thickness d. (d=D/N)

Two assumptions are used to simplify the calculation:

1. The k vector direction of the s and p modes for each layer is assumed to be at the same angle.
2. Boundary effects are ignored.

Equations are here derived to find the amplitude and polarization state of each Floquet order in a particular layer, given the amplitude and polarization state of each of the diffraction orders in the layer before it.

The main steps are:

Step 1: Find the form of the dielectric tensor of PPD.

Step 2: Find the form of the electric field of normal incident light.

Step 3: Plug the electric field of light and dielectric tensor of PPD into the Helmholtz equation and discretize the first-order differential equation to yield the propagation equations.

2.2 Detailed calculation steps

2.2.1 Step 1: Find the form of the dielectric tensor of PPD.

In this paper, P_x is defined as positive if, along the +x direction, the director is rotated such that the right-hand rule of the thumb points to the +z direction. P_x is defined as negative if, along the +x direction, the director is rotated such that the right-hand rule of the thumb points to the -z direction.

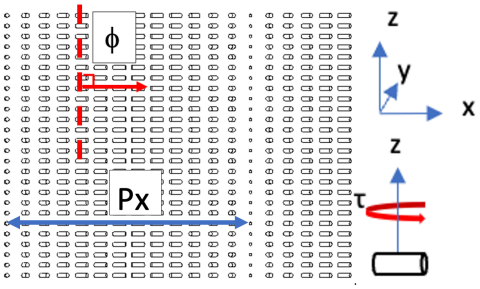

Figure 5. 2-dimensional PPD picture. The grating vector is the red arrow. The grating vector angle Φ=+90 ° with respect to the +z axis. According to our definition, P_x is positive.

The angle τ describes the director rotation about the z-axis due to P_x.

$$\tau = \pi\left(\frac{x}{p_x}\right)$$

According to the Floquet condition, the wave vector of the ℓth order can be written as,

$$\vec{k_l} = \vec{k_0} - l\vec{k_{grating}}$$

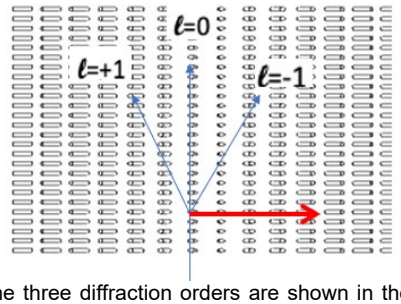

Figure 6. The three diffraction orders are shown in the figure with light normally incident to the PPD. The grating vector is the red arrow.

As in [9], the dielectric tensor can be written in two terms. One term unrelated to the pitch is ε_a, and the other is the perturbation term $\Delta\varepsilon$.

$$\varepsilon = \varepsilon_a + \Delta\varepsilon = \varepsilon_a + \frac{\Delta N^2}{4} M_+ exp\left(ik_x{}^{grating} \cdot x\right) + \frac{\Delta N^2}{4} M_- exp\left(-ik_x{}^{grating} \cdot x\right) \quad (2)$$

Where i is the imaginary unit, and M matrices, ΔN^2 and ε_a are defined as,

$$M_+ = \begin{bmatrix} 1 & -i & 0 \\ -i & -1 & 0 \\ 0 & 0 & 0 \end{bmatrix} \quad M_- = \begin{bmatrix} 1 & i & 0 \\ i & -1 & 0 \\ 0 & 0 & 0 \end{bmatrix} \quad \Delta N^2 = n_e{}^2 - n_o{}^2$$

$$\varepsilon_a = \begin{bmatrix} n_o{}^2 + \frac{\Delta N^2}{2} & 0 & 0 \\ 0 & n_o{}^2 + \frac{\Delta N^2}{2} & 0 \\ 0 & 0 & 0 \end{bmatrix}$$

2.2.2 Step2: Find the form of the electric field of normal incident light.

Following the same method in[7], the total electric field is then, for all existing orders of ℓ,

$$\vec{E_{tot}} = \sum_l S_l \exp\left\{i\left(l \cdot k_x{}^{grating} \cdot x - k_z{}^{l=0} \cdot z\right)\right\}$$

where $\vec{k_l}$ is the wave vector for the lth order, $\vec{k_0}$ is the incident wave vector, $\vec{k_{grating}}$ is the grating vector. The approximation that the direction of the k vector for the s and p modes is the same is used. S_ℓ is defined as,

$$S_l = A_l R(\theta_l) \begin{bmatrix} J_{sl} \\ J_{pl} \\ 0 \end{bmatrix}_k$$

21680159, 2024, 1, Downloaded from https://sid.onlinelibrary.wiley.com/doi/10.1002/sdtp.17479 by Kent State University Standing Orders, Wiley Online Library on [06/08/2024]. See the Terms and Conditions (https://onlinelibrary.wiley.com/terms-and-conditions) on Wiley Online Library for rules of use; OA articles are governed by the applicable Creative Commons License

A_l is the amplitude for the lth order, J are components of the Jones vector and $R(\theta_l)$ is the rotation matrix from the axis of k vector to the +z lab axis.

$$R(\theta_l) = \begin{pmatrix} \cos(\theta_l) & 0 & -\sin(\theta_l) \\ 0 & 1 & 0 \\ \sin(\theta_l) & 0 & \cos(\theta_l) \end{pmatrix}$$

$$\begin{pmatrix} J_{inx} \\ J_{iny} \\ J_{iny} \end{pmatrix} = R(\theta_l) \begin{pmatrix} J_{ins} \\ J_{inp} \\ 0 \end{pmatrix}$$

2.2.3 Step 3: Plug the electric field of light and dielectric tensor of PPD into the Helmholtz equation and discretize the first-order differential equation to yield the propagation equations.

Plug the form of the dielectric tensor (2) and the form of the electric field in grating (3) in the wave equation (1), after some calculation similar to [7],

$$\frac{dS_l}{dz} = \frac{-i \cdot \pi \cdot \lambda}{2n_{ave}} \left\{ -\frac{2l^2}{{P_x}^2} S_l + \frac{\Delta N^2 M_- S_{l+1}}{2\lambda^2} + \frac{\Delta N^2 M_+ S_{l-1}}{2\lambda^2} \right\}$$

Now we can discretize the equation by thinking of the total thickness D consisting of a very large number N of layers of thickness d.

$$d = \frac{D}{N} = \frac{\lambda}{N \cdot \Delta n}$$

With PPD, there only exist -1st, 0th and +1st diffraction orders.

With a similar calculation in [7], we have the beam propagation equation,

$$\begin{aligned} S_{-1\,order}^{layer\,n+1} &= S_{-1\,order}^{layer\,n} + i(ES_{-1\,order}^{layer\,n} - CM_- S_{0\,order}^{layer\,n}) \\ S_{0\,order}^{layer\,n+1} &= S_{0\,order}^{layer\,n} - iC(M_+ S_{-1\,order}^{layer\,n} + M_- S_{+1\,order}^{layer\,n}) \\ S_{+1\,order}^{layer\,n+1} &= S_{+1\,order}^{layer\,n} + i(ES_{+1\,order}^{layer\,n} - CM_- S_{0\,order}^{layer\,n}) \end{aligned} \quad (3)$$

Where C and E are defined as,

$$C = \frac{\pi d \Delta N^2}{4 n_{ave} \lambda} \qquad E = \frac{\pi d \lambda}{n_{ave} {P_x}^2}$$

Note that there is no relation between the complex amplitude of the -1st and +1st diffraction orders, meaning energy only transfers between adjacent orders.

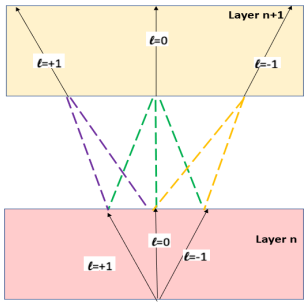

Figure 7. The diffraction order in layer n+1 can be defined by the adjacent diffraction orders in layer n.

3. Use 'ρ' parameter to explain the drop of efficiency for large diffraction angle PPD deflector

Define 'ρ' as the ratio of E over C.

$$\rho = \frac{E}{C} = \frac{2\lambda^2}{({P_x}^2 \cdot {n_{ave}}' \cdot \Delta n)}$$

where,

$${n_{ave}}' = \frac{(n_e + n_o)}{2}$$

So (3) becomes,

$$\begin{aligned} S_{-1\,order}^{layer\,n+1} - S_{-1\,order}^{layer\,n} &= iC(\rho S_{-1\,order}^{layer\,n} - M_- S_{0\,order}^{layer\,n}) \\ S_{0\,order}^{layer\,n+1} - S_{0\,order}^{layer\,n} &= iC(M_+ S_{-1\,order}^{layer\,n} + M_- S_{+1\,order}^{layer\,n}) \\ S_{+1\,order}^{layer\,n+1} - S_{+1\,order}^{layer\,n} &= iC(\rho S_{+1\,order}^{layer\,n} - M_- S_{0\,order}^{layer\,n}) \end{aligned} \quad (4)$$

Note on the left side is the change of complex amplitude from layer to layer. The two terms on the right side are generally out of phase. A larger ρ value is expected make the phase changes more as the light propagates.

To understand the effect of the ρ parameter, below we plot the intensity and phase when the light propagates through the small(10 degrees) and large(40 degrees) diffraction angle PPD for RCP incidence.

With RCP incident light, only 0th and -1st diffraction order exists[7]. We can clearly see the phase of the -1st order is changes much more for large diffraction angle PPD(yellow curve in figure 8(d)) than for small diffraction angle PPD(yellow curve in figure 8(c)).

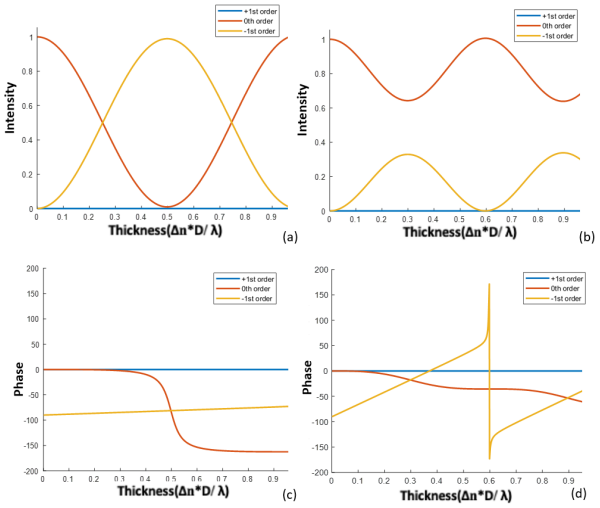

Figure 8. With birefringence=0.1 and wavelength=0.633 microns, incident polarization=RCP (a) intensity vs thickness for P_x=3.65 microns(10-degree deflector) (b) intensity vs thickness for P_x=0.98 microns(40-degree deflector) (c) phase vs thickness for P_x=3.65 microns(10-degree deflector) (b) phase vs thickness for P_x=0.98 microns.(40-degree deflector)

For a small diffraction angle PPD deflector, P_x is large, ρ is small, the -1st order will remain a constant phase related to the 0th order. When propagating from layer to layer, the change of complex amplitude from layer to layer adds constructively for the -1st order. As a result, the energy initially in the 0th order, can continuously

21680159, 2024, 1, Downloaded from https://sid.onlinelibrary.wiley.com/doi/10.1002/sdtp.17479 by Kent State University Standing Orders, Wiley Online Library on [06/08/2024]. See the Terms and Conditions (https://onlinelibrary.wiley.com/terms-and-conditions) on Wiley Online Library for rules of use; OA articles are governed by the applicable Creative Commons License

flowing into the -1st order before the half wave thickness and the intensity is expected to be high at half wave thickness.

However, for a large diffraction angle PPD deflector, as P_x decreases, ρ is not small, then as the light propagates, the phase of -1st diffraction order becomes unpredictable relative to the 0th order. Because the phase of the change of complex amplitude varies from layer to layer, the intensity can't increase continuously. If the phase of the -1st diffraction order is completely random, we can expect the intensity to be close to 0. This effect is referred to as "dephasing" in isotropic material study[5]. The dephasing of -1st order from 0th order prevents the energy from being transferred from 0th to -1st order, yielding low diffraction efficiency.

With LCP incidence, the analysis is unchanged, except the existing order being +1st and 0th order.

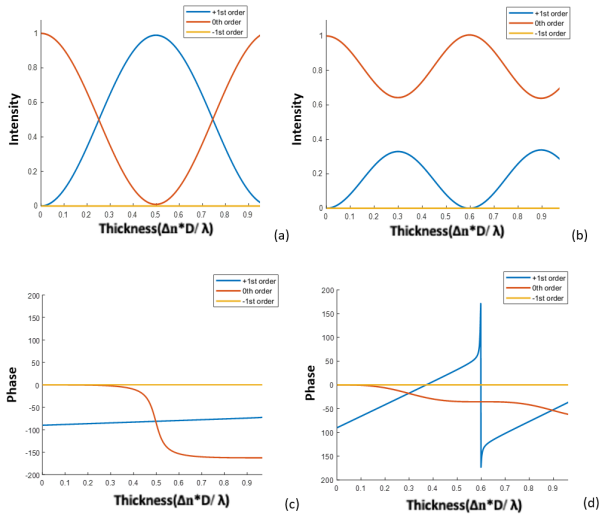

Figure 9. With birefringence=0.1 and wavelength=0.633 microns, incident polarization=LCP (a) intensity vs thickness for P_x=3.65 microns. (b) intensity vs thickness for P_x=0.98 microns. (c) phase vs thickness for P_x=3.65 microns (b) phase vs thickness for P_x=0.98 microns.

4. Conclusion and discussion

In this work, using the method in [7], the limitation of the PPD in diffraction efficiency is explained analytically. When the diffraction angle is large, the +/-1st order has a rapidly varying phase relative to the 0th order, leading to low diffraction efficiency.

The physical explanation of this rapidly varying phase of the +/-1st order with respect to the 0th order for the case of a large 'ρ' value can be visualized by considering the simplified ray trace diagrams in Figure 10.

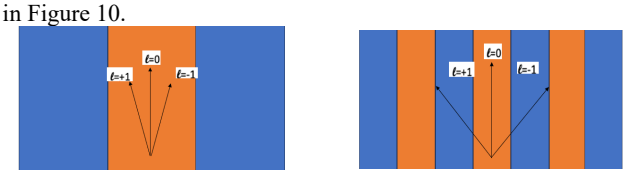

Figure 10. Figure shows an incident light beam to a periodic media and the resulting diffracted orders for small angle deflector PPD(left) and larger angle deflector(right).

When the diffraction angle is small, the P_x is large, the +/-1st order does not see a large index variation when the beam propagates through the PPD. The dephasing effect is small, and the energy can transfer to +/-1st order, yielding high diffraction efficiency.

When the diffraction angle is large, the P_x is small, the +/-1st order sees a large index variation when the beam propagates through the PPD. The dephasing of the +/-1st order from 0th order prevents the energy from being transferred from 0th to +/-1st order, yielding low diffraction efficiency.

6. Acknowledgements

This work was funded by Meta Reality Labs.

7. References

[1] S. Pancharatnam, "Generalized theory of interference and its applications," Proc. Indian Acad. Sci. A, 44 398 (1956).

[2] Escuti, Michael J., Jihwan Kim, and Michael W. Kudenov. "Controlling light with geometric-phase holograms." Optics and Photonics News 27.2 (2016): 22-29.

[3] Sarik R. Nersisyan, Nelson V. Tabiryan, Diane M. Steeves, and Brian R. Kimball, "The Promise of Diffractive Waveplates," Optics & Photonics News 21(3), 40-45 (2010)

[4] Lu Lu, Taha Masood, and Barry Silverstein, "Toward Lighter, Thinner AR/VR Systems," Optics & Photonics News 32(7), 42-47 (2021)

[5] Gaylord, T., and M. Moharam. "Planar dielectric grating diffraction theories." Applied Physics B 28 (1982): 1-14.

[6] Oh, Chulwoo & Escuti, Michael. (2007). Numerical analysis of polarization gratings using the finite-difference time-domain method. Physical Review A.

[7] Fenglin Xi and Philip Bos, "Intuitive understanding of the connection between Pancharatnam–Berry optical beam deflectors and polarization volume holograms," Appl. Opt. 62, 1845-1852 (2023)

[8] Xiayu Feng, Lu Lu, Oleg Yaroshchuk, and Philip Bos, "Closer look at transmissive polarization volume holograms: geometry, physics, and experimental validation," Appl. Opt. 60, 580-592 (2021)

[9] Tomoyuki Sasaki, Kenta Miura, Osamu Hanaizumi, Akira Emoto, and Hiroshi Ono, "Coupled-wave analysis of vector holograms: effects of modulation depth of anisotropic phase retardation," Appl. Opt. 49, 5205-5211 (2010)

21680159, 2024, 1, Downloaded from https://sid.onlinelibrary.wiley.com/doi/10.1002/sdtp.17479 by Kent State University Standing Orders, Wiley Online Library on [06/08/2024]. See the Terms and Conditions (https://onlinelibrary.wiley.com/terms-and-conditions) on Wiley Online Library for rules of use; OA articles are governed by the applicable Creative Commons License